Additional experiments should be performed to reveal the effects of roughness, mean radius of rotation, and distance of initial section from axis of rotation on increase in pressure loss.

NOTATION

$Re = wd/\nu$, Reynolds number for flow rate; $Re_\omega = \omega d^2/2\nu$, Reynolds number for circumferential velocity; $N_c = \omega^2 l d/w^2$, criterion taking into account the effect of centrifugal forces; $N_C = Re\sqrt{\omega d/w}$, criterion taking into account the effect of Coriolis forces, $\overline{\lambda}$, ratio of the coefficients of friction for rotating and stationary channels; w, flow rate; d, channel diameter; ν, fluid viscosity; ω, angular velocity; l, channel length; R_{av}, mean radius of rotation of channel; λ, coefficient of friction; Re_{cr}, critical value of Reynolds number; $\overline{F}$, ratio of intensities of centrifugal and Coriolis forces; A, m, constants. Indices: cf, cp, centrifugal and centripetal flows.

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A NOTE ON THE THEORY OF THE RANK EFFECT

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The possibility of using the equations of motion of a viscous compressible gas in the Kasterin–Predvoditelev form, to describe the vortex effect, is discussed.

In the theory of vortical flows there exist two basically different approaches. The first is connected with the name of L. Prandtl who assumed that the origination of a vortex takes place in a thin boundary layer, i.e., the nature of origination of the vortex is due to viscosity.

The second approach was advanced by Felix Klein who showed the possibility of origination of a vortex in an ideal liquid as a consequence of discontinuities occurring in the basic hydrodynamic parameters. The idea of Klein was developed by Kasterin [1] who obtained the equations of the vortex field in an ideal liquid, taking into account a discontinuous variation of the hydrodynamic velocity vector. A molecular-kinetic generalization of the equations of Kasterin for a viscous liquid was carried out by Predvoditelev [2, 3, 5].

We recall that to obtain the equations of motion of a viscous liquid in the form of the Navier–Stokes equations, Maxwell had to introduce two hypotheses [4]:

1. In a physically infinitely small volume the transport velocities of two colliding molecules are equal.

2. Between molecules of the gas there act forces of repulsion whose magnitude is inversely proportional to the fifth degree of the distance between the molecules.

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Predvoditelev proposed a new hypotheses relative to the transport velocities of two colliding molecules. He introduced the following conditions for the transition to a continuum:

1. There must exist an infinitely small volume for which we can calculate average quantities. The vector of transport velocity of the center of gravity of this volume has the components u, v, w. Then for a molecule of the first kind to be able to collide with a molecule of the second kind, the following conditions must be fulfilled:

$$u_2 < u < u_1;\ v_2 < v < v_1;\ w_2 < w < w_1.$$

2. The random motion of the medium is such that gradients of the transport velocity **V** in an appreciable manner vary along the length of a "free run" of molecules.

On the basis of these assumptions Predvoditelev obtained the following equations of hydrodynamics:

$$\frac{\partial \mathbf{V}}{\partial t} + (1-\beta)\,\mathrm{grad}\,\frac{\mathbf{V}^2}{2} + (1-\beta)\,[\mathrm{rot}\,\mathbf{V}\times\mathbf{V}] - \beta\mathbf{V}\,\mathrm{div}\,\mathbf{V} = -\frac{1}{\rho}\,\mathrm{grad}\,P + \left\{\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial \mathbf{V}}{\partial x} + \mathrm{grad}\,v_x\right)\right]\right.$$
$$\left. + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial \mathbf{V}}{\partial y} + \mathrm{grad}\,v_y\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial \mathbf{V}}{\partial z} + \mathrm{grad}\,v_z\right)\right] + \mathrm{grad}\left[\left(\eta_V - \frac{2}{3}\,\eta\right)\mathrm{div}\,\mathbf{V}\right]\right\}\frac{1}{\rho}, \tag{1}$$

where v_x, v_y, v_z are the components of velocity in a Cartesian coordinate system, η_V is the coefficient of volume viscosity, and β is the parameter of nonideal solidity (Predvoditelev's criterion);

$$|\beta| = \frac{3}{2}\,\mathrm{Kn}\cdot\mathrm{M} = \mathrm{Pd}.$$

Here Kn is the Knudsen number and M is the Mach number.

Equations (1) for $\beta = 0$, i.e., when the transport velocities of the approaching molecules are the same, are transformed into the Navier–Stokes equations. For $\beta = 2$, Eqs. (1) are transformed into the equations of Kasterin, if the viscosity and the product **V** div **V** are put equal to zero.

We apply the equations of a compressible gas in the Predvoditelev–Kasterin form to describe the Rank vortex effect.

For a steady-state axisymmetric motion in a cylindrical coordinate system, if we exclude the volume viscosity in conformity with the Stokes hypothesis, we have

$$(1-\beta)\left(\rho u\,\frac{\partial u}{\partial r} + \rho w\,\frac{\partial u}{\partial z} - \frac{\rho v^2}{r}\right) - \beta\rho u\,\mathrm{div}\,\mathbf{V} = -\frac{\partial P}{\partial r} +$$
$$+ \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\,\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{\mu}{3}\,\frac{\partial}{\partial r}\,\mathrm{div}\,\mathbf{V}, \tag{2}$$

$$(1-\beta)\left[\rho u\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \rho w\,\frac{\partial v}{\partial z}\right] - \beta\rho v\,\mathrm{div}\,\mathbf{V} = \mu\left(\Delta v - \frac{v}{r^2}\right), \tag{3}$$

$$(1-\beta)\left(\rho u\,\frac{\partial w}{\partial r} + \rho w\,\frac{\partial w}{\partial z}\right) - \beta\rho w\,\mathrm{div}\,\mathbf{V} = -\frac{\partial P}{\partial z} + \mu\Delta w + \frac{\mu}{3}\,\frac{\partial}{\partial z}\,\mathrm{div}\,\mathbf{V}. \tag{4}$$

In the works cited, concerned with a theoretical investigation of the Rank effect, such models of vortical flows were considered in which the distribution of the circumferential component of velocity was studied, while the axial motion of the gas was assumed to be known [6-9].

We shall consider the flow of a gas in a device [9] depicted schematically in Fig. 1.

It is assumed that in the zone $r_1 \le r \le a$ the axial velocity component $w = 0$, while into the zone $0 \le r \le r_1$ the gas is supplied uniformly over the entire length L of the device. From the condition that expenditure of the gas is constant and the continuity equation div $(\rho\mathbf{V}) = 0$ we easily obtain [9] the following expressions for the radial velocity component:

$$\rho u = -\frac{nb}{2\pi r}\rho_0 v_0 \quad (r_1 \leqslant r \leqslant a), \tag{5}$$

$$\rho u = -\frac{nb}{2\pi r_1}\rho_0 v_0 \frac{r}{r_1} \quad (0 \leqslant r \leqslant r_1), \tag{6}$$

where n is the number of nozzles, and b is the width of the nozzle.

The expressions (5) and (6) can be used in the subsequent calculations instead of the continuity equation. We further assume that the circumferential component of velocity v does not depend on z and that we can neglect the term V div V in Eq. (3). As was shown in [5], this is equivalent to neglecting the velocity of volume dilatation at the point of discontinuity of the hydrodynamic velocity vector. In this case, Eq. (3) for the circumferential velocity component can be integrated independently. Its solution will be the function

$$v = \frac{B}{r}\int r \exp\left[\int \frac{(1-\beta)\rho u}{\mu}\,dr\right]dr + \frac{D}{r}, \tag{7}$$

where B and D are arbitrary constants.

We introduce the dimensionless quantities

$$r' = \frac{r}{r_1};\quad v' = \frac{v}{v_0} \tag{8}$$

(primes are omitted in the following).

With the expressions (5), (6) and the boundary conditions v(0) = 0, $v(a/r_1) = 1$, as well as the joining conditions on the zone boundaries, taken into account, we obtain the following solutions for the circumferential velocity component:

$$v = \frac{c_1}{r} + \frac{c_2}{r^{\mathrm{Re}^*-1}} \quad \left(1 \leqslant r \leqslant k = \frac{a}{r_1};\ \mathrm{Re}^* \neq 2\right), \tag{9}$$

$$v = \frac{c_3}{r}\left[1 - \exp\left(-\frac{\mathrm{Re}^*}{2}r^2\right)\right] \quad (0 \leqslant r \leqslant 1). \tag{10}$$

If Re* = 2, then

$$v = \frac{c_4 \ln r}{r} + \frac{c_5}{r} \quad (1 \leqslant r \leqslant k), \tag{11}$$

$$v = \frac{c_6}{r}\left[1 - \exp(-r^2)\right] \quad (0 \leqslant r \leqslant 1). \tag{12}$$

Here $\mathrm{Re}^* = \mathrm{Re}(1-\beta)$, $\mathrm{Re} = nb\rho_0 v_0/2\pi\mu$, while the coefficients $c_1 - c_6$ are calculated from the expressions

$$c_1 = k - c_2 k^{2-\mathrm{Re}^*},$$

$$c_2 = \frac{2\,\mathrm{Re}^*}{2-\mathrm{Re}^*}\exp\left(-\frac{\mathrm{Re}^*}{2}\right)c_3,$$

$$c_3 = \frac{k(2-\mathrm{Re}^*)(k^{2-\mathrm{Re}^*}-1)}{(2-\mathrm{Re}^*)\left[1-\exp\left(-\frac{\mathrm{Re}^*}{2}\right)\right] + 2\mathrm{Re}^*(k^{2-\mathrm{Re}^*}-1)\exp\left(-\frac{\mathrm{Re}^*}{2}\right)},$$

$$c_4 = \frac{4}{e}c_6,\quad c_5 = \left(1 - \frac{1}{e}\right)c_6,$$

$$c_6 = k\left[1 - \frac{1}{e} + \frac{4}{e}\ln k\right]^{-1}.$$

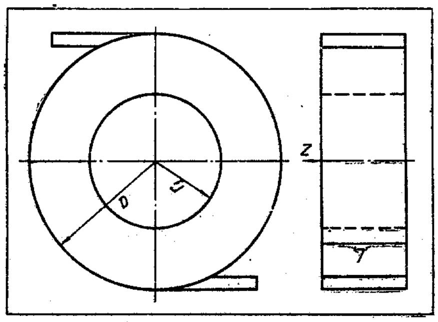

Fig. 1. Scheme of vortex chamber.

If we put $\beta = 0$, then the expressions (9)-(12) are transformed into the solutions obtained in [6-8]. Subsequently these results were repeated by Gol'dshtik [9].

The first terms of the expressions (9) and (10) determine the circumferential velocity for a potential flow, while the remaining terms reflect the vortex structure of the flow due to viscous forces. Analyzing the expressions (9) and (10) for $\beta = 0$ and $Re \to \infty$, it is not difficult to note that $v \to const/r$, signifies the degeneration of the vortical flow into a potential flow. This fact is paradoxical. Indeed, as the number Re increases, the vortical flow must become stronger. Since Re is calculated from the characteristic circumferential velocity component (in the given case, from v_0), and it cannot become potential. The experimental results of [7, 10] confirm this.

The presence of $\beta \neq 0$ allows us to eliminate the possibility of a transition, when $Re \to \infty$, to a potential flow in the expressions (9) and (10). We put, for example, $(1-\beta) = const/Re$. Then the second term in the expression (9) will be finite. The parameter β can be a function of the Reynolds number and the form of this function must be determined by experiment.

In the case $\beta = 0$ the expressions (11) and (12) exist only for Re = 2, and this is not understandable from a physical viewpoint. If it turns out that the function $\beta(Re)$ is multivalued, then for $Re^* = 2$ we obtain the second class of solution of the problem being considered.

It should be noted that by taking into account the terms V div V in the equations of motion we can improve the functional dependence of the parameter β on the number Re.

NOTATION

u, v, w, radial, tangential and axial components of velocity in the cylindrical system of coordinates r, φ, z; V, vector of total velocity; ρ, mean gas density; v_0, nozzle outflow velocity; ρ_0, mean gas density at the nozzle exit; μ, dynamic coefficient of viscosity; P, pressure; a, radius of outer zone of flow; r_1, radius of inner zone of flow; Δ, Laplace operator.

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